Understanding the Connections between Students’ Natural Way of Reasoning and Mathematical Ways of Reasoning

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Motivation for Study: All humans make inductive inferences constantly and without formal instruction. Developmental research demonstrates that even very young children have some remarkably sophisticated inferential strategies. Yet, in contrast to the ubiquitous nature of, and children’s facility with, inductive reasoning, deductive reasoning typically occurs only in restricted contexts (e.g., mathematics classrooms) and is often a difficult skill for children to acquire. In the world outside of the mathematics classroom, it is common and reasonable to develop facts and ideas via empirical generalizations and causal theories. Thence, the overarching question motivating this project is whether we can elaborate strategies used for inductive reasoning in both non-mathematical and mathematical contexts, and whether those inductive strategies can be built upon to help students develop deductive, mathematical arguments.

Abstract: Despite the growing emphasis on proof in school mathematics, research continues to paint a bleak picture of students’ abilities to reason mathematically. In contrast, recent cognitive science research has revealed surprising strengths in children’s ability to reason in non-mathematical contexts. Accordingly, we examined the connections between students’ reasoning in mathematical and natural contexts. Our study was designed to measure how convincing the students found different forms of evidence in three areas: numerical relations, geometry, and natural science. We found two themes consistent across areas: that dissimilar examples were always more convincing than similar, and that irrelevant examples were always less convincing than other examples. The evidence also indicates that while typical examples for numerical relations and geometry were generally more convincing, in natural science the atypical examples were slightly less convincing. Furthermore, in geometry and natural science, dissimilar examples were more convincing than similar, while in numerical relations the results were less conclusive.

Methods:
- 47 Middle school mathematics students
  - 1 6th-grade student, 26 7th-grade students, and 20 8th-grade students
  - Paper and pencil surveys
  - Statements paired with 5 different types of evidence
  - Numerical relations (natural numbers, 1-6)
  - Geometry (three pairs of triangles and rectangles)
  - Natural science (three sets of three animals, 3 pairs of ants, wasps, and butterflies)
- Ranking in relation to each other
- Each survey contained irrelevant, random, and deductive evidence items, as well as a single set of paired theme items: similar and dissimilar evidence; typical and atypical evidence; and 1 example and 5 examples.
- Analysis across the three areas and 9 qualities of evidence
- Task items were within the capacity of middle school-aged students
- More examples were consistently better than fewer
- Irrelevant examples were consistently weaker than relevant
- Results for similarity and typicality were mixed
- Some evidence of domain differences
- Evidence quality with most confidence (one examples versus five examples)
- Fewer than 2/3 of the students showed a distinction for mathematical items
- 80% showed the distinction for animal items

Conclusions:
- Tasks were within the capacity of middle school-aged students
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Future Directions:
- Diagnostic students’ representations of categories, similarities, and typicalities in mathematical domains
- Examine middle-school, non-STEM college, and STEM graduate students
- Empirically measure similarities and typicalities in numerical relations and geometry (this work was based on our intuition and best guesses)

Example Triad Similarity Measurement Item:
Which of the following is more similar to 101.
- 99
- 105

Abstract:

**SIMILAR vs. DISSIMILAR vs. IRRELEVANT**

**NUMERICAL RELATIONS**

<table>
<thead>
<tr>
<th>Quality</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIMILAR</td>
<td>Three added these sets of consecutive numbers that were very similar: 1 + 2 + 3 = 6 2 + 3 + 4 = 9 3 + 4 + 5 = 12</td>
</tr>
<tr>
<td>DISSIMILAR</td>
<td>Three added these sets of consecutive numbers that were very different: 1 + 2 + 3 = 6 2 + 3 + 4 = 9 3 + 4 + 5 = 12</td>
</tr>
<tr>
<td>IRRELEVANT</td>
<td>Three added these sets of consecutive numbers that were unrelated: 1 + 2 + 3 = 6 2 + 3 + 4 = 9 3 + 4 + 5 = 12</td>
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**GEOMETRY**

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<td>SIMILAR</td>
<td>Three added these sets of shapes that were very similar: triangle + square + circle = 3</td>
</tr>
<tr>
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**NATURAL SCIENCE**

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<td>Three added these sets of animals that were very similar: ants + bees + butterflies = 3</td>
</tr>
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